Dark state cooling of atoms by superfluid immersion

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We propose and analyse a scheme to cool atoms in an optical lattice to ultra-low temperatures within a Bloch band, and away from commensurate filling. The protocol is inspired by ideas from dark state laser cooling, but replaces electronic states with motional levels, and spontaneous emission of photons by emission of phonons into a Bose-Einstein condensate, in which the lattice is immersed. In our model, achievable temperatures correspond to a small fraction of the Bloch band width, and are much lower than the reservoir temperature.

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Fundamental advances in atomic physics are often linked to the development of novel cooling methods, as illustrated by laser and evaporative cooling, which led to the recent realization of degenerate Bose- and Fermi-gases [1]. This has further led to the achievement of strongly correlated atomic ensembles in the lowest Bloch band of an optical lattice [2, 3, 4, 5]. However, in order to realise some of the most interesting condensed matter phases predicted for lattice Hamiltonians, even better purification of the motional state is necessary, in particular for atoms in a partially filled Bloch band [5]. Here we propose a method for cooling atoms to mean energies much smaller than the width of the lowest Bloch band $4J^0$. In our setup (c.f. Fig. 1a) lattice atoms a are excited to the first Bloch band via a Raman laser pulse except when they occupy Bloch states with quasi-momentum close to zero – so-called dark states. The lattice is immersed in a Bose-Einstein Condensate (BEC) of a different atomic species b, so that the atoms can subsequently decay back to the lowest band via collisional interactions with the BEC reservoir [6, 7]. Thus the atom is recycled to the lowest band by emission of a phonon – or more precisely, a Bogoliubov excitation [6]. By repeated application of laser excitation and "spontaneous emission", cooling into the dark state region of quasi-momenta is achieved without loss of atoms [19] (c.f. Fig. 1b).

This method is inspired by the seminal Kasevich-Chu scheme [8] for sub-recoil laser cooling [9, 10], but replaces internal atomic states by Bloch band excitations, and spontaneously emitted photons by phonons. Our scheme thus operates on a much smaller energy scale than laser cooling, with correspondingly lower temperatures. This method can also be seen as a form of sympathetic cooling, where the energy is removed by phonons with energies equal to the Bloch band separation. Such phonon modes will initially be in the vacuum state, giving an effective $T=0$ reservoir, and allowing temperatures *significantly lower than the BEC reservoir temperature*, in contrast to standard sympathetic cooling. The ability to switch the collisional interactions via Feshbach resonances [11] enables us to study the cooling scenario in the weakly interacting gas, creating strongly correlated phases by ramping up the interaction in a final step.

On a formal level our cooling scheme can be written as the iterative application of a map,

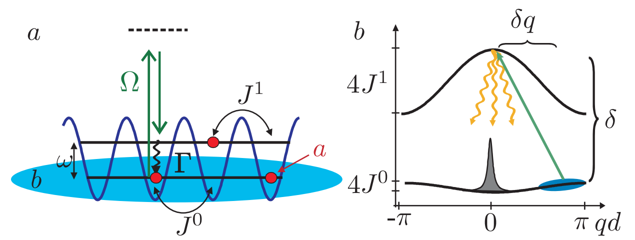

FIG. 1: (a) Cooling setup: Atoms a in an optical lattice are coupled to the first excited motional state via a Raman process, and decay to the ground motional state due to collisional interactions with a BEC of species b in which the lattice is immersed. Tunnelling between neighbouring sites with amplitude J^α gives rise to Bloch bands. (b) Momentum space picture: Atoms with higher quasi-momentum q are excited to the upper Bloch band, and decay to random quasi-momentum states. After several cycles, atoms a collect in a dark state region near $q=0$ with low excitation probability.

$$\mathcal{M}_j : \hat{\rho}_j \rightarrow \hat{\rho}_{j+1} \equiv \left(\hat{\mathcal{D}} \circ \hat{E}_j\right) \hat{\rho}_j, \tag{1}$$

where the density operator $\hat{\rho}_j$ describes the atoms in the lowest Bloch band before the jth step. Each step consists of two parts: the coherent laser excitation $\hat{E}_j$ of lattice atoms a, and the dissipative decay $\hat{D}$ returning atoms to the lowest band via coupling to the reservoir b (Fig. 1). To achieve the best possible cooling, differently shaped excitation pulses $\hat{E}_j$, $j=0,\ldots,N_p-1$ are applied, and this sequence is repeated, with $\hat{E}_j = \hat{E}_{j \bmod N_p}$. This repeated application of the map corresponds to a purification of the density operator, starting from an initial mixed state (e.g., a thermal distribution) towards a pure state (at zero temperature, $T=0$). In order to find appropriate forms of the Raman pulses and the action of $\hat{D}$, we analyse the dynamics of the lattice atoms and their interaction with the reservoir gas.

We consider a one dimensional model for the motion of atoms a, which is readily generalised to higher dimensions. Including Raman coupling, the Hamiltonian is $\hat{H}_a = \hat{H}_0 + \hat{H}_I$,

with

$$\hat{H}_0 = \sum_{q,\alpha} \varepsilon_q^\alpha \left(\hat{A}_q^\alpha\right)^\dagger \hat{A}_q^\alpha + (\omega - \delta) \sum_q \left(\hat{A}_q^1\right)^\dagger \hat{A}_q^1 + \frac{\Omega}{2} \sum_q \left[\left(\hat{A}_q^1\right)^\dagger \hat{A}_{q-\delta q}^0 + \text{h.c.}\right], \quad (2)$$

$$\hat{H}_I = \frac{1}{2} \sum_{i,\alpha} U^{\alpha\alpha} \hat{n}_i^\alpha (\hat{n}_i^\alpha - 1) + U^{10} \sum_i \hat{n}_i^1 \hat{n}_i^0. \quad (3)$$

Here, $\hat{A}_q^\alpha$ and $(\hat{A}_q^\alpha)^\dagger$ are annihilation and creation operators for quasi-momentum q in Bloch band $\alpha \in \{0,1\}$, satisfying Bose or Fermi (anti-)commutation relations. The kinetic energy is $\varepsilon_q^\alpha = -2J^\alpha \cos(qd)$, where d is the lattice spacing, J^α are the tunnelling amplitudes (with $J^0 > 0$ and $J^1 < 0$, see Fig. 1b), and ω is the band separation. The effective Rabi frequency $\Omega = \Omega_R \int dx \exp(-i\delta q x) w^1(x) w^0(x)$, where Ω_R is the two photon Rabi frequency as a function of time during the pulse and $w^\alpha(x)$ the Wannier functions. The Hamiltonian is a two-band model, written in a rotating frame with Raman detuning δ, and δq denotes the momentum transfer. We choose units $\hbar = k_B = 1$, where k_B is the Boltzmann constant. The parameters Ω, δ and δq will change during the pulse sequence, but be constant during a given pulse j. Onsite interactions between lattice atoms are represented by $\hat{H}_I$, with $\hat{n}_i^\alpha$ the number operator for atoms in site i and band α, and $U^{\alpha\alpha'}$ the associated onsite energy shifts [2].

The density-density interaction between lattice atoms a and a three dimensional BEC reservoir b, which gives rise to the decay $\hat{\mathcal{D}}$, is described by the Hamiltonian [6]

$$\hat{H}_{\text{int}} = \sum_{\alpha,\alpha'} \sum_{\mathbf{k},q} \left(G_{\alpha,\alpha'}^{\mathbf{k}} \hat{b}_{\mathbf{k}} \left(\hat{A}_q^\alpha\right)^\dagger \hat{A}_{q-k}^{\alpha'} + \text{h.c.}\right). \quad (4)$$

Here, the operator $\hat{b}_{\mathbf{k}}^\dagger$ creates a Bogoliubov excitation with momentum $\mathbf{k} = (k, k_y, k_z)$, and neglecting overlap of Wannier functions in different lattice sites, the coupling $G_{\alpha,\alpha'}^{\mathbf{k}} \approx g_{ab}(S(\mathbf{k})\rho_b/V)^{1/2} \int d^3x e^{i\mathbf{kx}} w^\alpha(\mathbf{x}) w^{\alpha'}(\mathbf{x})$. The strength of the inter-species contact interaction is denoted g_{ab}, ρ_b and V are the density and volume of the BEC reservoir, and $S(\mathbf{k})$ is the static structure factor [1]. For excitation energies less than the chemical potential μ, excitations are sound waves for which $S(\mathbf{k})$ is strongly suppressed, and $S(\mathbf{k}) \to 0$ as $|\mathbf{k}| \to 0$ [1]. For energies larger than μ, excitations are in the particle-like sector of the spectrum, with much larger $S(\mathbf{k}) \to 1$. Here we will typically have $4J^0 < \mu < \omega$, so that decay between bands is induced by particle-like excitations with strong coupling, but collisional processes between the reservoir and atoms in the lowest Bloch band are suppressed. In close analogy to Ref. [6, 7] we derive a master equation for the reduced system density operator, describing the decay between bands in the Born-Markov approximation [20]. The associated Liouvillian is $\mathcal{L}[\rho] = \sum_k \Gamma_k \left(2c_k\rho c_k^\dagger - c_k^\dagger c_k \rho - \rho c_k^\dagger c_k\right)/2$. The momentum k along the lattice axis is bounded by $|k| \le \sqrt{2m_b\omega}$ due to energy conservation, where m_b is the mass of atoms b, and the jump operators c_k are defined as $c_k = \sum_q A_{q-k,0}^\dagger A_{q,1}$. The resulting decay rates Γ_k for spontaneous emission of a phonon with momentum k projected on the axis of the lattice can be written explicitly for deep lattices, where $\omega \gg |J^1|, J^0$ and the individual lattice sites can be approximated as harmonic oscillator potentials. We find $\Gamma_k = g_{ab}^2 \rho_b m_a a_0^2 k^2 \exp(-a_0^2 k^2/2)/2L$, with a_0 the size of the ground state in each lattice site, m_a the mass of atoms a, and L the length of the 1D lattice. We denote the total decay rate from the excited band by $\Gamma = \sum_k \Gamma_k$. We consider a situation where dissipation is switched off during the excitation step, so that $\hat{E}_j$ and $\hat{D}$ occur separately. This can be achieved e.g., by tuning the collisional interaction so that $g_{ab} \approx 0$. We can read the action of $\hat{E}_j$ and $\hat{D}$ for a given step j from the master equation.

We first illustrate the cooling process for a single lattice atom, designing a sequence of Raman laser pulses, where the j-th pulse excites the atom with initial quasi-momentum q in the lowest band to the first excited band with probability $P_j(q)$. We require $P_j(q) = 0$ for $q \approx 0$, but $P_j(q) \to 1$ for states with high quasi-momentum (c.f. Fig. 1b). In analogy with Raman cooling schemes in free space [8] we choose square pulses with duration $\tau_j = \pi/\Omega_j$, for which $P_j(q) = \sin^2(\sqrt{\delta_{q+\delta q_j}^2 + \Omega_j^2}\tau_j/2)\Omega_j^2/(\delta_{q+\delta q_j}^2 + \Omega_j^2)$, with the effective detuning $\delta_{q+\delta q_j} \equiv \omega + \varepsilon_{q+\delta q_j}^1 - \varepsilon_q^0 - \delta_j$.

An example of an efficient pulse sequence is shown in Fig. 2a-c. We begin with an intense laser pulse which resonantly excites atoms with momentum $q \sim \pi/d$ around the edges of the Brillouin zone (Fig. 2a). The subsequent pulses move the resonant transition closer to $q = 0$ by adjusting the momentum transfer δq_j and Raman detuning δ_j (Fig. 2b). In order to prevent excitation of atoms with $q = 0$, we decrease Ω and increase the pulse duration, τ for the later pulses, each time achieving $P_j(q = 0) = 0$. To resolve the transition we must always have $\Omega \ll 8|J^1|$, and therefore $\tau \gg \pi/8|J^1|$. Note that it is the value of J^1 and not J^0 that gives the resolution of the excitation. However, the relationship between J^1 and J^0 is fixed by the lattice depth (e.g., for the parameters used in Figs. 2–4, $V_0 = 10\omega_R$, we have $J^0 = 0.019\omega_R$ and $J^1 = -0.25\omega_R$). By combining a sequence of 5 pulses, one can efficiently excite most atoms with $|q| > 0$, as shown in Fig. 2c. Note that the pulse sequence has been carefully designed to avoid significant population outside the lowest two bands.

To quantitatively analyze the cooling process we numerically simulate the time evolution of the density operator $\hat{\rho}$ using quantum trajectories [12] starting from a completely mixed state in the lowest band ($T \gg 4J^0$), with $\sim 10^5$ trajectories. During the cooling process, the momentum distribution develops a sharp peak near $q = 0$ after very few iterations as shown in Fig. 2d. In Fig. 2e we plot the temperature of the state as defined by $k_B T/2 = \pi^2 \sin^2(\Delta q\, d)(J^0)^2/\omega_R$, where Δq is the half-width of the momentum distribution at e^{-1} of the maximum value. We find excellent agreement between our

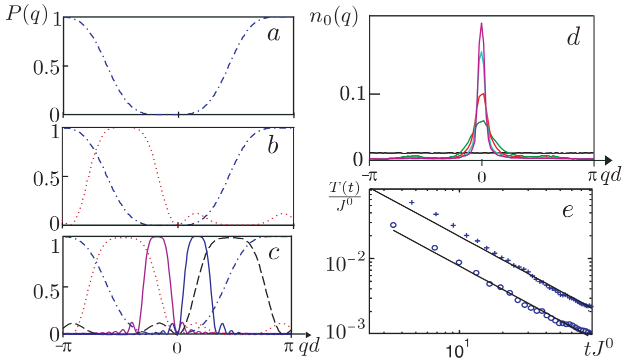

FIG. 2: (a)-(c) Excitation probability $P-j(q)$ for a sequence of $N_p=5$ pulses: first (a-c, dash-dot), second (b-c, dotted), and the remaining pulses (c). Parameters used: $\Omega=(27.9,13.7,13.7,2.37,2.37)J^0$, $\delta q=(0.16,-1.75,1.75,-2.63,2.63)/d$, $(\delta-\omega)=-(28.4,25.8,25.8,24.7,24.7)J^0$ for the different pulses. (d) Successive narrowing of the momentum distribution in the lower Bloch band after $0,1,3,5$ and 10 cooling cycles from numerical simulations ($M=101$ lattice sites) based on the pulse sequence in (a)-(c). We choose parameters for ^{87}Rb in the lattice and ^{23}Na in the reservoir, with $\Gamma=53J^0$ from $m_a/m_b=3.73$, $\rho_b=5\times10^{14}\mathrm{cm}^{-3}$ and scattering length $a_{ab}=14$nm. $V_0=10\omega_R$, and $\omega_R=2\pi\times3.8$kHz. (e) Temperature vs. time for a single atom: crosses and circles denote numerical, solid lines analytical results based on Lévy statistics. Pulse sequences for circles: same as in (d); crosses: $N_p=3$ pulses with $\Omega=(32.6,7.9,7.9)$, $\delta qd=(0.31,2.12,-2.12)$, $(\delta-\omega)=-(28.4,25.3,25.3)J^0$.

numerical calculations and analytical results obtained with Lévy statistics [8, 10] as a function of time. The latter predicts a final temperature $T\propto t^{-1}$ for square pulses, as shown in Fig. 2e. For a zero-temperature reservoir, and parameters as in the caption for Fig. 2, we reach temperatures $T\sim2\times10^{-3}J^0$ in time $t_fJ^0\sim30$.

Finite temperature T_b in the reservoir can lead to sympathetic heating of lattice atoms a by absorption of thermal phonons, as described by $\hat{H}_{\rm int}$. However, this process is forbidden by energy and momentum conservation, provided $J^0<\sqrt{\mu\omega_R m_a/(2m_b)}/\pi$. In detail, energy conservation requires $c|\mathbf{k}|=\varepsilon^0_q-\varepsilon^0_{q'}$ and conservation of momentum along the lattice direction leads to $k=q-q'$ ($|k|\le|\mathbf{k}|$), where the atom a is scattered from quasi-momentum $q\approx0\to q'$ by absorption of a phonon with momentum $\mathbf{k}$, and c is the sound velocity in the BEC. These conditions cannot be fulfilled unless the above inequality is violated. Higher order processes involving two or more thermal phonons will be small. In numerical simulations we also checked that the cooling protocol is insensitive to small timing errors. Whilst in the above protocol we have switched off the decay during application of $\hat{E}_j$, we can leave decay switched on, provided that $\Gamma\ll1/\tau\ll|J^1|$. This will restrict the length of the pulses that can be applied, thus slowing the cooling process.

The cooling scheme can be readily adapted to many bosons

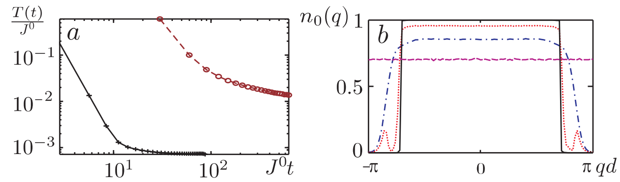

FIG. 3: Numerical simulation of the QBME: (a) Temperature as a function of cooling time for bosons (crosses) and fermions (circles). (b) Momentum distribution in band $\alpha=0$ for fermions after 0 (dashed), 1 (dash-dot), 2 (dotted), and 20 (solid) cooling cycles, each with $N_p=4$ pulses. Parameters used: Bosons: As for Fig.2a-c, but $N=51$ particles. Fermions: $N=71$, $M=101$, Blackman pulses with $\tau J^0=(1.78,1.78,6.8,6.8)$, $\delta qd=(0.19,-0.19,0.75,-0.75)$, $(\omega-\delta)=(28.4,28.4,27.9,27.9)J^0$; $V_0=10\omega_R$, $\omega_R=2\pi\times6$kHz, $m_a/m_b=1.74$ and $\Gamma=52.6J^0$.

or fermions. For bosons, we assume that the collisional interaction between atoms a is tuned to zero ($\hat{H}_I\to0$). We work out the efficiency of the cooling protocol by deriving a quantum Boltzmann master equation (QBME) [13], which describes transitions between classical configurations of atoms occupying momentum states in the Bloch bands, $\mathbf{m}=[\{\mathrm{m}^0_q\}_q,\{\mathrm{m}^1_q\}_q]$, where m^α_q is the occupation number of quasi-momentum state q in band α. This is derived from the master equation by projection of the density operator ρ onto diagonal elements, $\mathcal{P}\hat{\rho}\mathcal{P}=\sum_{\mathbf{m}}w_{\mathbf{m}}|\mathbf{m}\rangle\langle\mathbf{m}|$, neglecting off-diagonal coherences. For the excitation step, $\hat{E}_j$, the evolution is computed exactly from the excitation probability $P_j(q)$, and for the decay, $\hat{D}$, we obtain

$$\dot{w}_{\mathbf{m}}=\sum_{k,q}\Gamma_k\left[\mathrm{m}^0_{q-k}(1\pm\mathrm{m}^1_q)w_{\mathbf{m}'}-\mathrm{m}^1_q(1\pm\mathrm{m}^0_{q-k})w_{\mathbf{m}}\right],$$

where $\mathbf{m}'=\mathbf{m}-\mathbf{e}_{q-k,q}$ is the resulting configuration when a particle with quasi-momentum q in the upper band decays to the lower band with new quasi-momentum $q-k$, i.e., $\mathbf{e}_{q-k,q}$ is a configuration vector with $\mathrm{m}^0_{q-k}=1$, $\mathrm{m}^1_q=-1$ and all other entries zero. The upper (lower) signs are for bosons (fermions). The approximation inherent in neglecting off-diagonal coherences only plays a role during the decay step, where these coherences couple to the occupation probabilities. We remark that an *exact* physical realisation of the QBME can be obtained by modulating the lattice depth after each excitation step, randomising the off-diagonal elements [14].

Fig. 3a shows the decrease in temperature as a function of time for bosons and fermions, obtained from monte carlo simulations of the QBME [13]. For bosons, we use the same excitation pulse sequence as for a single atom in Fig. 2. The cooling process in this case outperforms that for a single atom, reaching low temperatures on shorter times due to bosonic enhancement (here we compute temperature as for a single atom, but fitting a Gaussian to the $q\neq0$ momentum distribution). For fermions, the pulse sequence must be changed to create a dark state region of quasi-momenta with $|q|<q_F$, where q_F is the Fermi momentum, in order to cool towards a $T=0$ Fermi distribution. In this case, time-square pulses are

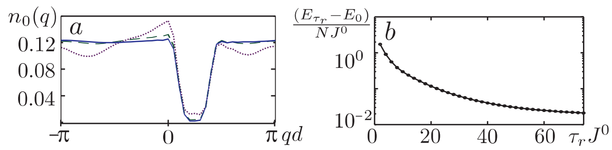

FIG. 4: (a) Momentum distribution in the lowest Bloch band from numerical simulations after a single excitation pulse beginning with equal occupation, for interaction strengths $U^{\alpha\alpha'} = (0, 0.1, 0.3)J^0$ (solid, dashed, and dotted lines). Parameters used: $N = 5$ atoms in $M = 41$ lattice sites $\delta qd = 1$, $\Omega = 1.05J^0$, $(\delta - \omega) = 27.9J^0$, $V0 = 10\omega_R$, $\omega_R = 2\pi \times 3.8$kHz. (b) Energy E_{τ_r} of the state obtained by beginning in the $U^{00} = 0$ ground state with $N = 10$, $M = 21$, and ramping the interaction strength as from $U^{00} \approx 0$ to $U^{00} \approx 20J^0$ in time τ_r, as $U^{00} = 20J^0(1 - \{1 + \exp[(t - \tau_r/2)/(\tau_r/10)]\}^{-1})$.

no longer efficient as there is a large secondary peak in $P_j(q)$ (see Fig. 2b), and we instead use Blackman pulses [8], which approach $P_j(q) = 0$ monotonically. The momentum distribution develops the expected shape of a cold Fermi distribution after very few iterations (Fig. 3b) (we compute temperatures for fermions by fitting a Fermi distribution to these results).

Our model predicts that the temperature will always decrease with increasing cooling time. Experimental imperfections will, in practice, give rise to decoherence and heating (e.g., from spontaneous emissions [2], collisions with background gases, scattering multiple phonons). One assumption made above for bosons was that the interaction between lattice atoms a is approximately zero. Using time dependent DMRG methods [15] we computed the population remaining in the lowest band after an excitation pulse with interactions present (Fig. 4a). For $U^{\alpha\alpha'} \ll 1/\tau \ll |J^1|$ there is no significant change in the excitation profile and the above conclusions should not change.

Finally, after the cooling we adiabatically ramp up the interaction strength to produce an interacting system, assuming that the system is decoupled from the reservoir b. This is illustrated in Fig. 4 for the case of ramping from a non-interacting to a hard-core Bose lattice gas in 1D (Tonks gas). We compute the evolution from a Bose-Hubbard model (H_a with no Raman coupling, and only the lowest band), again using time dependent DMRG methods [15]. The energy deposited during the ramp of the interaction strength is plotted as a function of the ramp time τ_r, and for ramping times on the order of $10/J^0$, we observe negligible heating within the Bloch band. Numerical tests using example excited states also showed that the energy difference between different initial states is not significantly increased during the ramp.

In summary, filtering quasi-momentum states in the lowest Bloch band of an optical lattice and recycling them by "spontaneous emission" of phonons combine to give a cooling scheme producing temperatures a small fraction of the lowest Bloch band width. These temperatures are a necessary step towards the realisation of strongly correlated quantum phases not currently accessible in optical lattice experiments.

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[20] We neglect effects due to reabsorption, which are discussed in the context of laser cooling [18]. These effects can be suppressed, e.g., by evaporative cooling of the reservoir [7].